Sensitivity of Electric Dipole Polarizability to Bulk Nuclear Properties

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Plan

Introduction

Hartree-Fock RPA

Results

Why study Giant Resonances?

- Determine properties of nuclear matter
- Helps to determine Energy Density Functional
- Nuclear Equations of State
- Better understand nuclear structure
- Understand astrophysical phenomena
  - Neutron Stars
  - Super Novae
  - R-Process
- Defense purposes

Giant Resonances

- What is Giant Resonance?
  - Classically we look at it as the oscillation of liquid drop
  - Quantum mechanically, it is the collective motion of individual nucleons in the nucleus: coherent sum of particle hole excitations
- Isovector Giant Dipole Resonance (IVGDR)
  - Dipole oscillation of protons against neutrons
- Experimental techniques
  - Photon excitation of nucleus

Theoretical Approach

- To determine the dynamics, we construct our Hamiltonian, and plug it into the Schrödinger equation:

$$H = T + V = \sum_i \frac{p_i^2}{2m_i} + \sum_{i<j} V_{i,j}$$

$$H \left|\Phi\right\rangle = E \left|\Phi\right\rangle$$

- That's too many terms!
- We use the mean field approximation
  - Best for large A nuclei

The Skyrme Interaction

- We use the Skyrme Interaction
  - An effective 10 parameter mean field potential
  - Contact term potential
  - Hundreds of published interactions, we use 33
- Has the form:

$$V_{i,j}^{NN} = t_0(1+x_0 P_{ij}^{\sigma})\delta(\vec{r}_i-\vec{r}_j)+\frac{1}{2}t_1(1+x_1 P_{ij}^{\sigma})\left[\overleftarrow{k}_{ij}^2\delta(\vec{r}_i-\vec{r}_j)\delta(\vec{r}_i-\vec{r}_j)\vec{k}_{ij}^2\right]+$$

$$t_2(1+x_2 P_{ik}^{\sigma})\overleftarrow{k}_{ij}^2\delta(\vec{r}_i-\vec{r}_j)\vec{k}_{ij}+\frac{1}{6}t_3(1+x_3 P_{ij}^{\sigma})\rho^{\alpha}\left(\frac{\vec{r}_i+\vec{r}_j}{2}\right)\delta(\vec{r}_i-\vec{r}_j)+$$

$$iW_0\overleftarrow{k}_{ij}\delta(\vec{r}_i-\vec{r}_j)(\sigma_i+\sigma_j)\vec{k}_{ij}$$

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Hartree-Fock Calculation

- We assume a slater determinant of single particle wave functions:

$$\Phi(x_1, x_2, ...x_n) = \frac{1}{\sqrt{n!}} \begin{vmatrix} \phi_1(x_1) & \phi_1(x_2) & \dots & \phi_1(x_n) \\ \phi_2(x_1) & \phi_2(x_2) & \dots & \phi_2(x_n) \\ \vdots & \vdots & \ddots & \vdots \\ \phi_n(x_1) & \phi_n(x_2) & \dots & \phi_n(x_n) \end{vmatrix}$$

- Now we use a variational approach to minimize the matrix element $\langle \Phi | H | \Phi \rangle$ thus giving our ground state

Hartree-Fock Equations

- We minimize $\langle\Phi| H |\Phi\rangle$ using a pertubutive method

$$\phi_i \rightarrow \phi_i + \delta\phi_i$$

- This gives rise to the HF equation for the ground state

$$-\frac{\hbar^2}{2m}\Delta\phi_i(\mathbf{r}) + \phi_i U_H(\mathbf{r}) - \int U_F(\mathbf{r},\mathbf{r'})\phi_i(\mathbf{r'})\, d^3\mathbf{r'} = e_i\phi_i(\mathbf{r})$$

$$U_H(\mathbf{r}) = \sum_j \int \phi_j^*(\mathbf{r'})V(\mathbf{r},\mathbf{r'})\phi_j(\mathbf{r'})\, d^3\mathbf{r'}$$

$$U_F(\mathbf{r},\mathbf{r'}) = \sum_j \phi_j^*(\mathbf{r'})V(\mathbf{r},\mathbf{r'})\phi_j(\mathbf{r})$$

Random Phase Approximation (RPA)

- Strength function given by:

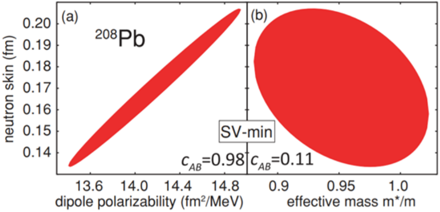
$$S(E) = \sum_n | \langle 0 | F_{LM} | n \rangle |^2 \delta(E - E_n)$$

$$F_{LM} = \frac{Z}{A} \sum_n f(r_n) Y_{LM}(n) - \frac{N}{A} \sum_p f(r_p) Y_{LM}(p)$$

- F_{LM} is the scattering operator, and $L = 1$ for IVGDR
- Moments $m_k = \int_0^\infty E^k S(E)\, dE$ are calculated from RPA
- We can investigate properties of nuclear resonance

$$\alpha_D = \frac{8\pi e^2}{9} m_{-1} \qquad E_{centroid} = \frac{m_1}{m_0}$$

Δr_{np} vs. α_D?



P. G. Reinhard, W. Nazarewicz PRL 81.051303 (2010)

- Take one interaction
- Vary one parameter at a time (J on left, reduced mass on right)
- Different interactions give different correlations

Range of our 33 Interactions:

Parameter	Minimum	Maximum
ρ_0	0.156	0.175
E/A	15.32	16.33
K	200.8	258.1
J	26.69	37.4
L	-29.38	129.33
K_{sym}	-401.43	159.57
Q_{sym}	11.73	883.05
K_τ, V	-498.11	-271.61
m*/m	0.56	1

Pearson Linear Correlation Coefficients

- Calculates how linearly correlated two sets, (X, Y) are
- Outliers from a set can significantly impact correlation
- $P = \frac{\mathrm{cov}(X,Y)}{\sigma_X \sigma_Y} = \frac{\sum_i (x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum_i (x_i - \bar{x})^2}\sqrt{\sum_i (y_i - \bar{y})^2}}$
- $P \in [-1, 1]$
  - $|P| > 0.8$ High correlation
  - $0.8 \geq |P| > 0.6$ Medium correlation
  - $0.6 \geq |P| > 0.5$ Low correlation
  - $0.5 \geq |P|$ No correlation

Calculated Correlations with α_D

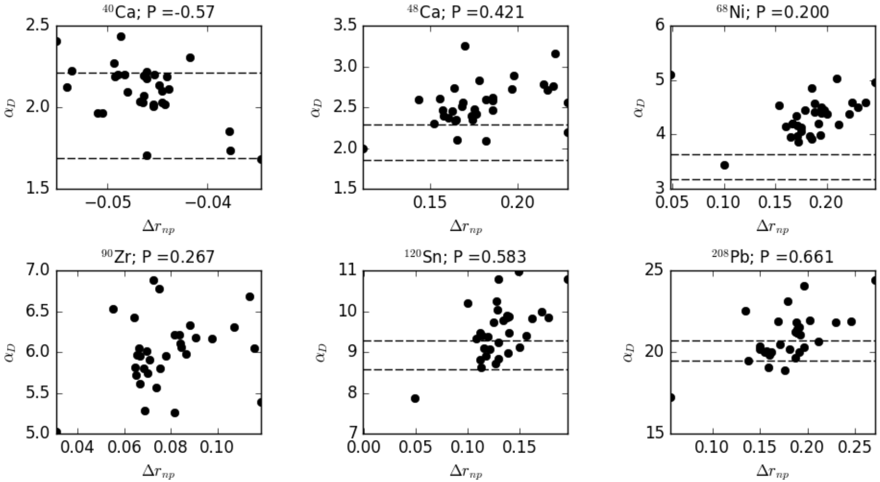
- To study sensitivity of α_D to BNM values we calculate Pearson linear correlation coefficients

Nucleus	Δr_{np}	J	L	Ksym	Qsym	$K\tau$, V
^{40}Ca	-0.57	-0.43	0.47	0.29	-0.61	-0.52
^{48}Ca	0.42	-0.30	0.39	0.26	-0.51	-0.43
^{68}Ni	0.20	0.37	0.70	0.59	-0.84	-0.60
^{90}Zr	0.34	0.25	0.52	0.42	-0.68	-0.44
^{120}Sn	0.58	0.26	0.60	0.51	-0.75	-0.46
^{208}Pb	0.67	0.19	0.61	0.57	-0.77	-0.42

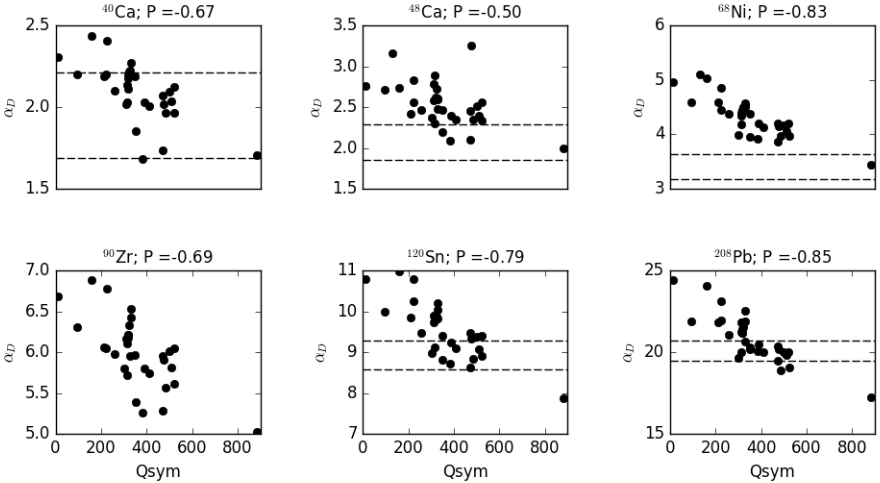
Looking at correlated properties

- Try to isolate values of BNM properties that systematically under or over estimate experiment
- Rule out certain interactions that have consistently bad predictions
- Very difficult given how low correlations are

α_D vs. Δr_{np}



α_D vs Q_{sym}



Conclusions

- Only weak-medium correlation between α_D and Δr_{np}
- α_D is only low-medium correlated with terms from equation of state besided Q_{sym}
- Cannot put reasonable bounds on Δr_{np} or BNM properties with current theory

Sources

- P. G. Reinhard and W. Nazarewicz PRL 81, 051303 (2010)
- J. Birkhan et al. PRL 118, 252501 (2017) (^{40,48}Ca Data)
- D.M. Rossi et al. PRL 111, 242503 (2013) (^{68}Ni Data)
- T. Hashimoto et al. PRL 92 031305 (2015) (^{120}Sn Data)
- A. Tamii et al. PRL 107, 062502 (2011) (^{208}Pb Data)

Acknowledgments

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Thank you!

Any questions?